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An Analysis of High Ionosphere Positive Ion Ram Probes

by

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ABSTRACT

An analysis of ion collection to a rocket borne ion probe is presented. The analysis is performed for both the planar and spherical geometry probes taking into account the presence of the ion sheath which surrounds the probe. From the above, the error introduced into the operation of the probe by the presence of the ion sheath is determined.

The analysis is applied to the high altitude ion probe which was flown on NASA Javelin 8.29 on May 19, 1965 as part of the Mother-Daughter project. Less than 2% error was attributed to the presence of the sheath for probe operation to an altitude of 500 km. At 500 km. the sheath size approaches the probe dimension, and the probe ceases to operate accourding to ram theory. From the analysis, design suggestions are made which theoretically enable an ion probe of this type to operate with less than 2% sheath error to an altitude of 1000 km.

CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

A planar-disc, retarding potential ion probe was flown on a rocket from Wallops Island, Virginia, on May 19, 1965. The primary purpose of the experiment was to measure ion densities between 300 and 1000 kilometers in the upper atmosphere. The data from this experiment was analyzed using ram theory as discussed by Whipple (1959). A complete description of this experiment is given in the scientific report by D. J. Hoffman (1966).

Ram theory assumes that the ion sheath which surrounds the probe has a negligible effect on its operation. For this reason, ram theory can be applied only when the sheath thickness is small in comparison with the probe dimensions. It is the purpose of this analysis to determine the error introduced by completely neglecting the effects of the ion sheath.

This analysis will be performed for both planar-disc geometry and the spherical geometry ion probes. It is hoped that a comparison in the sheath effects of these two geometries will give a better understanding into the relative advantage of using a planar geometry probe instead of a probe of curved geometry.

Finally, the planar-disc sheath analysis will be applied to the ion probe experiment in evaluating the error which is introduced by the use of ram theory. The analysis will serve as a basis for proposing probe modifications which would possibly improve the accuracy of its operation.

1.2 Historical Background

The Langmuir probe technique for measuring concentrations of charged particles and the temperature of electrons in discharge plasmas provides a valuable method for making such measurements from space vehicles . The use of such a probe in plasma research was originally conceived by Langmuir, and Mott-Smith (1926). Langmuir based his theory on thermodyamic equilibrium and a Maxwellian distribution of positive ion and electron energies. Experimentally, an electrode is inserted into a plasma, and the current to the electrode is measured as a function of the applied electrode voltage. From the resulting current-voltage characteristics, the charged particle energy distributions and densities are obtained.

The initial attempt to use a Langmuir probe on a space vehicle was made by Hok and Dow in 1946 as described by Hok, Spencer, and Dow (1953). Their experiment consisted of a rather large probe, in the form of a conic frustrum, mounted on the nose of a V-2 rocket, which was fired from White Sands, New Mexico. They considered several sources of error which were responsible for the rather poor results. It was concluded that the ambiguity of the probe geometry, the existence of negative ions, electron emission from the probe, the effect of the earth's magnetic field, and the air flow about the rocket caused considerable error in the measurements.

R. L. F. Boyd (1950) was the first experimenter to successfully employ the use of a controlling grid on a laboratory probe. The basic Langmuir probe was modified by placing a screen grid between the plasma and the collecting electrode. When the grid was biased at +48 volts, only electron current was collected. Positive ion current could be collected at a grid voltage of -60 volts.

By his experimental work, Boyd was able to reveal that the flux of electrons to the probe is not solely determined by the random thermal motion of the electrons as earlier theorized by Langmuir, but that the process is of a more complicated nature.

Additional grids were added by others, Hinteregger (1960), Hale (1961), and Bourdeau (1962) to suppress photo-electrons and secondary emission. The problems associated with fabricating a multiple grid configuration led to the development of the planar geometry gridded probe. This type of probe is frequently referred to as an ion trap.

Hinteregger (1960) successfully utilized a set of four grids to measure positive ion and electron densities, accounting for photo-electrons, secondary electrons, and negative ions to an altitude of 234 km. He realized that the accuracy of his experiment was dependent upon how well the vehicle potential could be estimated and how negligible were the sheath effects.

A four-grid ion probe was launched from Wallops Island, Virginia in 1960 on the scout vehicle (NASA, ST-II) as described by L. C. Hale (1961). The importance of this experiment was that Hanson (1962), using the ion probe data that was presented by Hale, deduced the existence of helium ions above 1200 km.

Bourdeau (1962) mounted a collecting electrode below a gridded aperture on the body of the explorer VIII Satellite. A grid biased at -15 volts was used to suppress photo electrons. As the Satellite rotated in the sun, no additional current was detected which implied that all photo electrons were being suppressed. From this retarding potential experiment, Bourdeau was able to obtain values of positive ion concentration, and the ratio of atomic helium to oxygen ion. He also confirmed the belief that 0+ ions predominate up to 800 km at night and up to at least 1500 km during midday, while helium ions predominate from 800 to at least 1200 km at night and from 1500 to at least 1800 km at midday.

The use of a spherical ion-trap on rockets is described by Sagalyn and Smiddy (1963). In their instrument, the collector and grid voltages were swept to give measurements of both electron and positive ion density. In theory they considered that the plasma exhibits a Maxwellian velocity distribution, the vehicle velocity to be a variable with respect to the most probable ion velocity, and the mean free path is large in comparison to the probe dimensions. Using this theory to analyze their experimental data, they found agreement within 20 per cent to simultaneous ionosonde data.

W. C. Knudsen (1966) used a planar-gridded ion trap to determine the ion concentration between 200-600 km. A discussion of the possible sources of error with a probe of this type is given by Knudsen. An analysis of the sheath effects is made using the assumption that the potential varies as $1/r$ and that the planar probe can be approximated by a hemi sphere. Knudsen shows the error introduced by sheath effects to be as great as 40 per cent for the cases considered.

CHAPTER II

DEVELOPMENT OF THE THEORY

2.1 Ion Current to a Retarding Potential Probe

The expression for the ion current, I, to a retarding potential probe has been previously derived and is thoroughly discussed by Whipple (1959). The assumptions made in deriving an expression for the ion current are that the sheath is thin and parallel to the face of the probe; the grids are equipotental surfaces; the grids and collector extend to infinity; the transparency of the grids is not dependent on the retarding potential or the angle of attack; all ions which are collected originate in the undisturbed plasma, and the ion mean free path length is large compared to the sheath and probe dimension.

A coordinate system is chosen fixed to the vehicle with the positive x-axis in the direction of vehicle motion. The relative velocity component, V_r, can be written as

$$V_r = C - V_R \tag{2.1}$$

where C is the thermal ion velocity in the x-direction, and V_R is the vehicle velocity. A Maxwellian velocity distribution is assumed and is given by

$$\frac{dN}{dC} = \frac{N}{\sqrt{2KT\pi/m}} \, C^{-\frac{mC^2}{2kT}} \tag{2.2}$$

where N = the neutral ion density

K = Boltzman's constant

T = ion temperature

m = ion mass.

The general expression for the collector current is given in equation (2.3)

$$I = \int \alpha e A V_r dN \qquad (2.3)$$

where A is the area of the collector plate, α is the grid transparency coefficient, and q is the ionic charge. Only those ions with a kinetic energy toward the rocket greater than the retarding potential barrier of the probe will be collected, that is an ion will be collected if:

$$\frac{1}{2} m V_r^2 > e\phi \quad .$$

This current can be written as

$$I = \frac{\alpha e A N}{\sqrt{2KT\pi/m}} \int_{-\infty}^{\sqrt{2e\phi/m}} V_r e^{-(V_r+V_R)^2 \frac{m}{2kT}} dV_r \qquad (2.4)$$

where ϕ is the retarding potential. Integrating equation (2.4) yields:

$$I = \alpha e A N V_R \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}(x) + \frac{\sqrt{\frac{2kT}{m\pi}}\, e^{-x^2}}{2V_R}\right) \qquad (2.5)$$

where $$x = \sqrt{\frac{m}{2kT}} \left(V_R - \sqrt{2e\phi/m}\right)$$

and $$ERF(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy \quad .$$

Equation (1.5) is the classical expression for ion current to a retarding potential ion probe, and is valid only when the thermal velocity of the ions is small in comparison to the rocket velocity.

2.2 Ram Current Expression

When the vehicle potential, V_o, goes negative so that nearly all ions in the path of the collector plate are collected, equation (2.5) reduces to the Ram current expression

$$I = A\alpha e N V_R \cos\theta \qquad (2.6)$$

where $V_R \cos\theta$ is the component of vehicle velocity normal to the collector plate.

2.3 Existence of an Ion Sheath

When an undisturbed body is placed in a plasma such as the ionosphere, electron and ion currents flow from the plasma to the body. The body will acquire a net negative charge or negative equilibrium potential due to the large thermal velocity of the electrons compared to the thermal velocity of the ions. The electron current to the body is produced only by those electrons energetic enough to overcome the equilibrium potential. At equilibrium, the total current to the body is zero with the electron current balanced by the positive ion current which is relatively independent of potential. Due to the small thermal velocity of the ions, the region which surrounds the body is deficit in electrons; hence, it will have a net positive charge which is referred to as the ion sheath.

2.4 Vehicle Potential

Since at equilibrium the total current to a body emersed in a neutral plasma is zero, a negative potential must exist on the body. Neglecting r.f. affects and photo-emission the vehicle potential, V_o, is given in equation (2.7) for a plane conducting body.

$$V_o = \frac{kT_e}{2e} \ln \frac{T_e M_+}{T_+ M_e} \tag{2.7}$$

T_e and T_+ are the electron and positive ion temperatures, while M_e and M_+ are their masses. Assuming an ionic constituent of 28 A.M.U. and thermodynamic equilibrium, equation (2.7) reduces to

$$V_o = 5.4 \frac{kT}{e} \tag{2.8}$$

where T is the kinetic gas temperature.

2.5 Idealized Sheath Model

As discussed by Jastrow and Pearse (1957), the charge distribution which surrounds a rapidly moving body emersed in a neutral plasma is Maxwellian and is given by expression (2.9):

$$\rho \approx q \left[N_+ - N_e e^{q\phi/kT}\right] \tag{2.9}$$

where ρ = net charge density which surrounds the body and is a function of position

q = electron charge

N_+ = neutral plasma positive ion density

N_e = neutral plasma electron density

ϕ = potential as a function of potential

T = electron temperature

k = Boltzman constant

which holds whenever

$$\sqrt{\frac{kT}{M_+}} \ll V_R \quad . \tag{2.10}$$

In a neutral plasma $N_+ = N_e$, hence expression (2.9) can be rewritten.

$$\rho \approx qN \left[1 - e^{\frac{q\phi}{kT}} \right] \tag{2.11}$$

From equation (2.11), it is observed that when the potential, ϕ, equals zero, the net charge density is also zero or the plasma is neutral. At the surface of the body the potential, ϕ, is equal to the vehicle potential. From expressions (2.8) and (2.11) the charge density at the surface of the probe can be determined and yields:

$$\rho \approx qN \left[1 - e^{-5.4} \right] \approx qn \quad .$$

Hence the charge density near the surface of the probe is nearly equal to the neutral plasma ion charge density. The exponential term in equation (2.11) is negligible for potentials, $\left|\phi\right| \geqslant \frac{kT}{2q}$. Figures (2.1) and (2.2) show the charge density about a rapidly moving probe as a function of potential. Since $V_o >> \frac{kT}{2q}$, $\frac{kT}{2q}$ is considered equal to zero in the idealized sheath model for a rapidly moving probe. Figure (2.2) shows the idealized sheath charge distribution. The sheath is assumed to have a sharply defined edge at $\phi = 0$.

2.6 Sheath Thickness

The sheath thickness can be determined using the idealized sheath model. Poisson's equation is solved and the following boundary conditions are utilized.

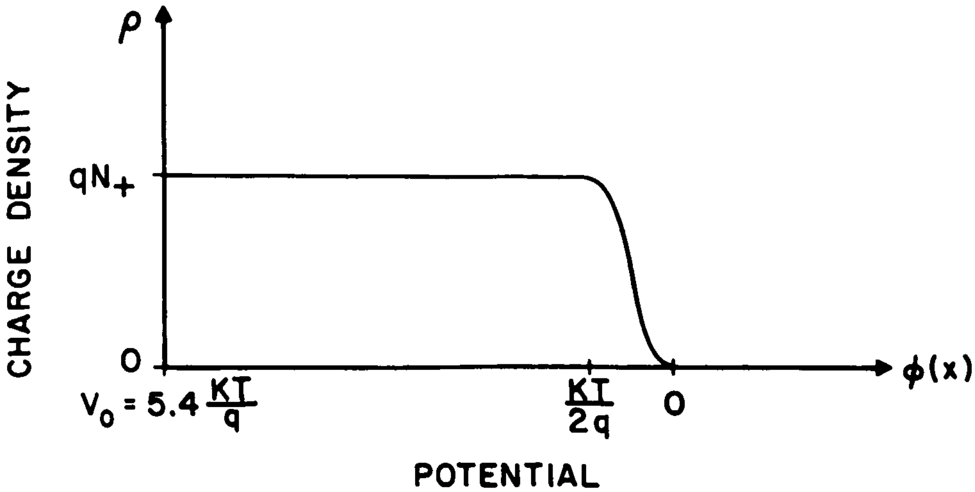
$$\phi = V_o \text{ at } x = 0$$

$$\phi = 0 \text{ at } x = d$$

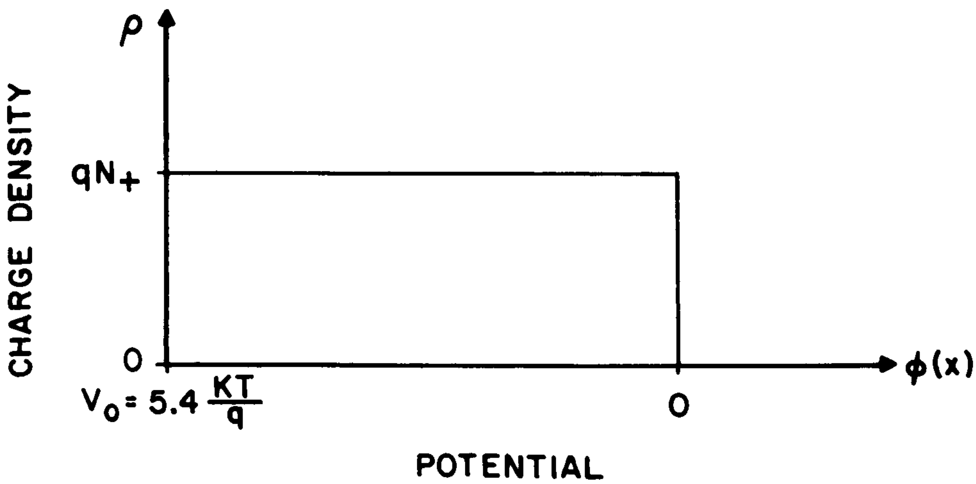
where d = sheath thickness.

In solving Poisson's equation the charge density is constant within the sheath and is equal to qN_+ as explained in Section (2.5).

The two cases of interest are the spherical and planar-disc probes. Jastrow and Pearse (1957) derive an expression for the sheath thickness of a spherical probe surrounded by a thin sheath.



ACTUAL SHEATH DENSITY
FIGURE 2.1



IDEALIZED SHEATH DENSITY
FIGURE 2.2

$$d = \sqrt{\frac{2V_o \epsilon_o}{Nq}} \tag{2.11}$$

A derivation for the sheath thickness of a planar-disc probe is given in Appendix A. The expression is valid for a thin sheath and is given by

$$d = \frac{\epsilon_o V_o}{qN\pi a} + \left[\left(\frac{2\epsilon_o V_o}{\pi a N q}\right)^2 - \frac{2\epsilon_o V_o}{Nq} \right]^{1/2} \tag{2.12}$$

where a = the radius of the disc. Equation (2.12) reduces to equation (2.13) for a very thin sheath.

$$d = \sqrt{\frac{2V_o \epsilon_o}{Nq}} \tag{2.13}$$

Equation (2.13) is identical to the expression that Jastrow and Pearse derived for the spherical probe.

CHAPTER III

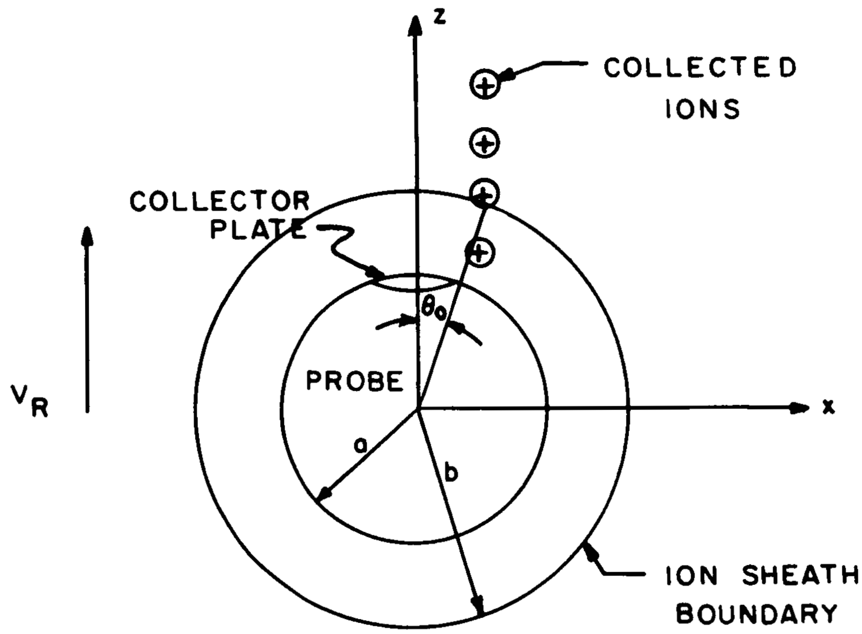
THEORY MODIFICATION

3.1 Ion Trajectory Analysis to a Spherical Probe

To obtain a first order sheath correction, simple plasma orbit theory will be applied. The motion of a single particle will be determined as it passes through the sheath to the collecting probe. By choosing a coordinate system which is fixed with respect to the probe, the ion enters the sheath at the probe velocity assuming that the ion thermal velocity is much less than the vehicle velocity. Figure (3.1) illustrates the concept of treating a single particle penetrating through the sheath to the probe. Once the motion of the single particle is determined, the actual current to the probe will be determined by superposing the motion of all the single particles.

Considering the earth's magnetic field as having a negligible effect on the motion of the ion, the forces acting on the particle are entirely due to the electric field which exists within the sheath. This field can be calculated by solving Poisson's equation for the potential. As mentioned previously, the charge density within the sheath is equal to the positive ion density of the medium. Thus Poisson's equation can be written in spherical coordinates,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = \rho/\epsilon_o = -\frac{Nq}{\epsilon_o}$$



THE SPHERICAL PROBE

FIGURE 3.1

which simplifies because of symmetry yielding:

$$\nabla^2 V = \frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = -\frac{\rho}{\epsilon_o} \quad .$$

The solution to Poisson's equation is given by:

$$V = A + \frac{B}{r} - \frac{\rho}{\epsilon_o} r^2 \quad . \tag{3.1}$$

The constants A and B are evaluated using the following two boundary conditions. The potential at the sheath edge, which corresponds to r=b, is taken to be zero while the potential at the surface of the probe, r=a, is equal to the probe potential ϕ_o. Equation (3.1) can be rewritten:

$$V = \mathcal{f}\left(\frac{b}{r}-1\right) - \frac{\rho}{6\epsilon_o} r^2 + \eta \tag{3.2}$$

where

$$\mathcal{f} = \frac{\left[\phi_o - \frac{\rho}{6\epsilon_o}(b^2-a^2)\right] a}{(b-a)}$$

and

$$\eta = \frac{\rho}{6\epsilon_o} b^2 \quad .$$

By taking the gradient of the potential the electric field can be obtained, and is written in equation (3.3).

$$E_r = -\nabla V = \int \frac{b}{r^2} + \frac{\rho r}{3\epsilon_o} \tag{3.3}$$

E_r is the radial electric field and is directed such that positive ions accelerate towards the probe. Equation (3.3) can be substituted into the Lorentz force equation and obtain the force acting on the particle as it moves through the sheath.

$$F_r = qE_r = m\frac{d^2r}{dt^2}\hat{a}_r \tag{3.4}$$

where m = the mass of the ion

$\hat{a}_r$ = the unit vector in the radial direction

Substituting equation (3.3) in for the electric field, equation (3.5) is written.

$$m\frac{d^2r}{dt^2} = q\left[\int \frac{b}{r^2} + \frac{\rho r}{3\epsilon_o}\right] \tag{3.5}$$

By rewriting the left hand side of equation (3.5)

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{dr}{dt}\right)^2\right]\frac{dt}{dr} = \frac{q}{m}\left[\frac{b}{r^2} + \frac{\rho r}{3\epsilon_o}\right]$$

and separating variables, both sides of the equation can be integrated, yielding:

$$\frac{m}{2}\left(\frac{dr}{dt}\right)^2 = q\left[-\rho\frac{b}{r} + \frac{\rho r^2}{6\epsilon_o}\right] + C_o \; . \qquad (3.6)$$

Since $\frac{dr}{dt}$ is the radial component of the ion velocity, the constant C_o can be evaluated. Equation (3.6) relates the kinetic energy of the particle to its position within the sheath, and is readily solved giving the velocity of the ion as a function of position.

In this analysis, the restriction made is that the sheath thickness is much smaller than the radius of the probe. This is a necessary condition which guarantees that the shape of the sheath is spherical about the probe. Remembering that this solution of Poisson's equation is valid only for a spherical sheath. For a thin sheath, it can be shown that:

$$\frac{q\rho r^2}{3m\epsilon_o} << \frac{2eb\rho}{mr} \; .$$

Hence, equation (3.6) can be rewritten neglecting the smaller of the two terms and solving for the velocity.

$$V_r = -\left[-\frac{2eb\rho}{mr} + C_1\right]^{1/2} \qquad (3.7)$$

The constant C_1 is evaluated at the sheath edge where the radial component of the ion velocity equals $-V_R \mathrm{Cos}\,\theta_o$. V_R is the rocket velocity and θ_o is the angle at which the ion enters the sheath. Restricting this analysis to small angles, $\cos\theta_o$ nearly equals unity. This condition restricts particle collection to the region near the center of the collecting surface. Under this condition, C_1 is evaluated:

$$C_1 = \frac{1}{2} V_R^{\;2} + \frac{2e\varphi}{m} .$$

The radial component of ion velocity is then given in equation (3.8)

$$V_r = \frac{dr}{dt} = -\left[\frac{2e\varphi}{m}\left(1-\frac{b}{r}\right) + V_R^{\;2}\right]^{-1/2} . \qquad (3.8)$$

By separating variables, equation (3.8) yields:

$$dt = -\left[\frac{2e\varphi}{m}\left(1-\frac{b}{r}\right) + V_R^{\;2}\right]^{-1/2} dr . \qquad (3.9)$$

By defining:

$$f(r) = -\left[\frac{2e\varphi}{m}\left(1-\frac{b}{r}\right) + V_R^{\;2}\right]^{-1/2}$$

equation (3.9) can be expanded about r=a in terms of f(a). The first three terms are given in equation (3.10).

$$f(r) = f(a) + (r-a)f'(a) + \frac{(r-a)^2}{2} f''(a) + \ldots \qquad (3.10)$$

where the primed superscript denotes the derivative with respect to r. Equation (3.9) can be rewritten by substituting in the expansion for f(r).

$$dt = f(a)dr + (r-a)f'(a)dr + \frac{(r-a)^2}{2} f''(a)dr + \ldots \qquad (3.11)$$

Term by term integration of equation (3.11) yields the total time required for an ion entering the sheath to be collected. The limits on t are from time equal zero to the total time T_P, and the limits on r are from r=b to where the ion is collected, r=a.

$$T_P = \int_o^{T_P} dt = f(a)\int_b^a dr + f'(a)\int_b^a (r-a)dr + f''(a)\int_b^a \frac{(r-a)^2}{2} dr + \ldots \qquad (3.12)$$

$$T_P = (a-b)f(a) - \frac{1}{2}(a-b)^2 f'(a) + \frac{1}{6}(a-b)^3 f''(a) + \ldots \qquad (3.13)$$

For a thin sheath, T_P is nearly equal to the first term of the expansion. Evaluating the thin sheath approximation for T_P :

$$T_P = -(a-b)\left[\frac{2e\int}{m}(1 - \frac{b}{a}) + V_R{}^2\right]^{-1/2} . \qquad (3.14)$$

The velocity which an ion enters the sheath, $-V_R$, can be split into two components; one is in the radial direction, V_r, and the other perpendicular to the radial direction, $V_\perp$, as shown in Figure 3.2. The radial component of ion velocity is given by $-V_R \mathrm{Cos}\,\theta_o$ which for small angles is nearly equal to $-V_R$. The perpendicular component of ion velocity is written as $-V_R \mathrm{Sin}\,\theta$ which for small angles can be approximated by $-V_R\,\theta_o$.

Since $V_\perp$ is perpendicular to the electric field, this component of velocity is conserved for small angles of attack. The total time, T_P, required for an ion entering the sheath to be collected is given in equation (3.14). Hence, the displacement of the ion in the ($\perp$) direction equals $V_\perp T_P$.

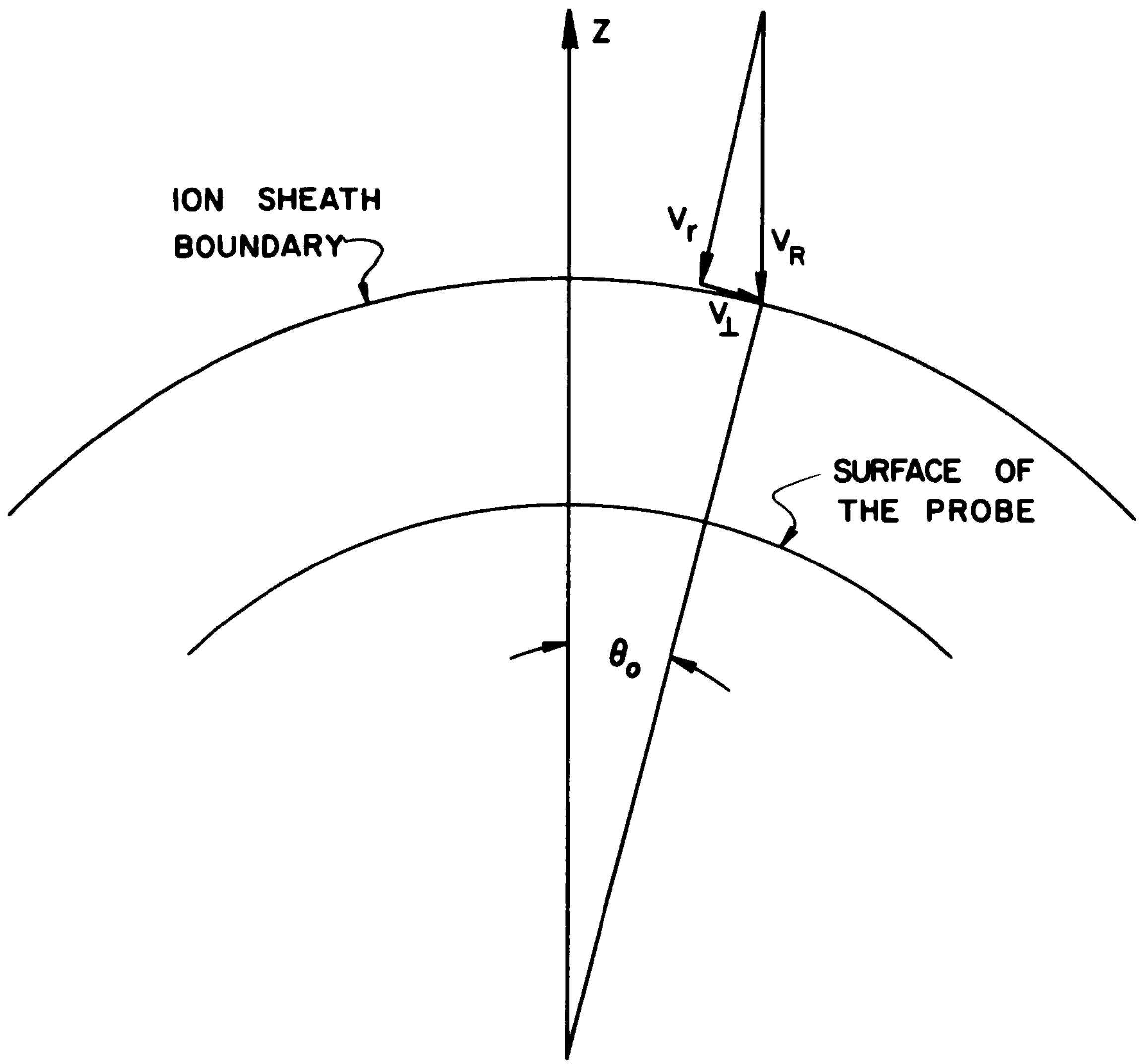
The ion displacement, R, in the radial direction can be written:

$$R = \int_o^{T_P} V_r(t)\,dt \quad .$$

For small angles of attack, R is nearly equal to the sheath thickness, (b-a), and is a displacement in the minus radial direction.

Initially, when the particle enters the sheath, the x-component of position is given by:

$$x_o = b\mathrm{Sin}\theta_o \quad . \tag{3.15}$$



COMPONENTS OF THE ION ENTRY VELOCITY

FIGURE 3.2

When the particle strkes the surface of the probe, the resulting x-component of its position is written in equation (3.16)

$$x_f = x_o + x_d \quad . \tag{3.16}$$

x_d is the total deflection in the x-direction of the ion traversing the sheath, and can be written by adding the two x-components of the perpendicular and radial displacement, $x_\perp$ and x_r,

$$x_d = x_\perp + x_r \tag{3.17}$$

where

$$x_\perp = V_\perp T_P \mathrm{Cos}\,\theta_o \tag{3.18}$$

and

$$x_r = (a-b)\mathrm{Sin}\,\theta_o \quad . \tag{3.19}$$

Thus, the total x-component of deflection can be written by substituting in equations (3.18) and (3.19) into equation (3.17).

$$x_d = V_\perp T_P \mathrm{Cos}\,\theta_o + (a-b)\mathrm{Sin}\,\theta_o \tag{3.20}$$

The final x-component of position given by equation (3.16) can be written.

$$x_f = V_\perp T_P \mathrm{Cos}\theta_o + a\mathrm{Sin}\theta_o \tag{3.21}$$

Equation (3.21) reduces for small angles of θ_o to

$$x_f = (V_R T_P + a)\theta_o \quad . \tag{3.22}$$

By subtracting the initial x-component of the particle's position from x_f, the deflection can be determined

$$\Delta x = \left[(b-a) - V_R T_P\right] \theta_o \quad . \tag{3.23}$$

The ratio of the deflection, Δx, to the initial x-component of position, x_o, is given in equation (3.24).

$$\frac{\Delta x}{x_o} = \frac{1}{b}\left[(b-a) - V_R T_P\right] \tag{3.24}$$

Substituting in for T_P, equation (3.24) can be rewritten.

$$\frac{\Delta x}{x_o} = \frac{(b-a)}{b}\left\{ 1 - V_R \left[\frac{2q\,\rho}{m\,6\epsilon_o}\,(b^2-a^2) - V_o + V_R^{\,2}\right]^{-1/2} \right\} \tag{3.25}$$

For a thin sheath approximation:

$$b^2 - a^2 \approx 2ad \quad . \tag{3.26}$$

Equation (2.11) gives an expression for the sheath thickness of a spherical probe in terms of the charge density. Solving equation (2.11) for the charge density, and together with relationship (3.26), equation (3.25) can be rewritten.

$$\frac{\Delta x}{x_o} = \frac{d}{b}\left\{1 - \left[1 - \frac{2qV_o}{mV_R^2}\left(1 + \frac{2}{3}\frac{a}{d}\right)\right]^{-1/2}\right\} \tag{3.27}$$

3.2 Error Analysis of the Spherical Probe

The positive ion current to the spherical probe is comprised of two components. One component is the ram current which is explained in sections (2.1) and (2.2) and is given by equation (2.6).

$$I_R = A\alpha eNV_R\cos\theta$$

Remembering that expression (2.6) is valid only when the vehicle velocity is much greater than the ion thermal velocity, and when the ion sheath is thin.

The other component of ion current is composed of only those ions which are deflected to the collector as they traverse the sheath. In Ram theory, this component is neglected. This component of current will be referred to as the sheath current and is given by equation (3.28).

$$I_s = qNV_R\, \alpha \left\{ \left[\left(\frac{\Delta x}{x_o} + 1\right) x_o \right]^2 \pi - \pi x_o^{\,2} \right\} \tag{3.28}$$

$\frac{\Delta x}{x_o}$ is given in equation (3.27), while x_o is taken to be the radius of the collector plate. Since $\pi x_o^{\,2}$ is equal to the area of the collector plate, A, the total ion current to the probe can be written:

$$I_T = I_R + I_S = qNV_R\; \alpha A\left(\frac{\Delta x}{x_o} + 1\right)^2 \tag{3.29}$$

By defining an effective area, A_{eff}, equation (3.29) can be rewritten:

$$I_T = qNV_R\, \alpha A_{eff}$$

where

$$A_{eff} = \pi \left[\left(\frac{\Delta x}{x_o} + 1\right) x_o \right]^2 .$$

The error introduced by using Ram theory and neglecting the sheath current is

$$\text{Per cent error} = E = \frac{A_{eff} - A}{A_{eff}} \times 100 \text{ per cent.} \tag{3.30}$$

Substituting in for both A and A_{eff}

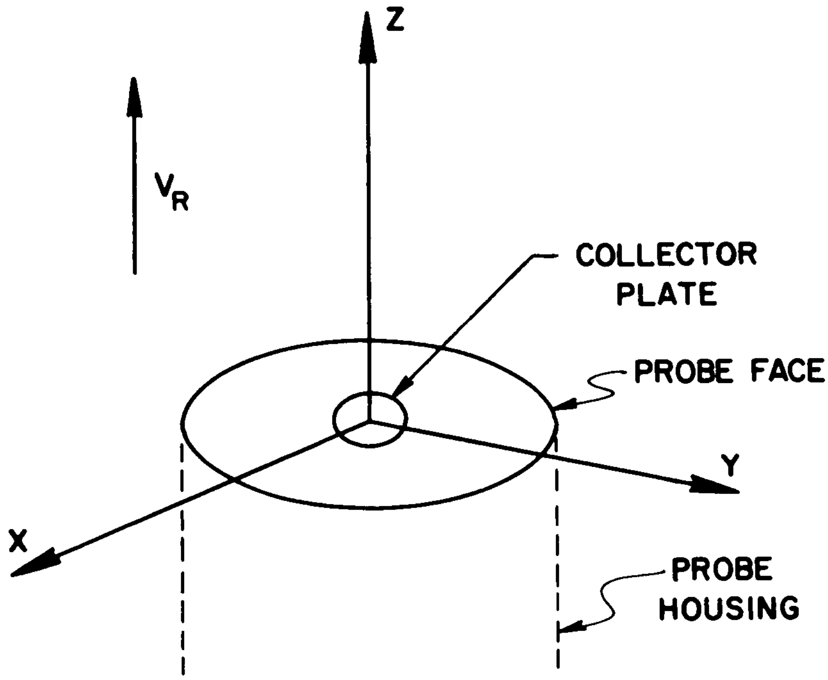
$$E = \left[1 - \left(\frac{\Delta x}{x_o} + 1\right)^{-2} \right] \times 100 \text{ per cent.} \qquad (3.31)$$

Quite interesting is the fact that the error is independent of the collector size as long as the collector plate is small when compared to the probe size. This is reasonable due to the fact that there is a corresponding increase in the ram current to the increase in sheath current brought about by the increase in collector size. Hence, the error can be determined for the spherical probe, requiring only the knowledge of a/d, and V_o/V_R^2, realizing that d/b can be determined from a/d.

3.3 Ion Trajectories to a Collecting Planar-Disc Probe

Treating the planar-disc probe in a manner similar to the spherical probe, the motion of a single particle as it passes through the sheath will be determined. A coordinate system is chosen fixed to the probe with the positive z-axis in the direction of vehicle motion as shown in Figure 3.3. The restrictions which were imposed on the spherical probe will be placed on this analysis. The analysis is valid only when collection is made very near the center of the probe, and the sheath thickness is small in comparison to the probe dimension. It should also be pointed out that the problem will be treated using the idealized sheath model as explained in Section (2.5).

The potential of a planar-disc is obtained by solving Poisson's equation. The solution of Poisson's equation in



THE PLANAR - DISC PROBE

FIGURE 3.3

cylindrical coordinates is given by equation (3.32).

$$\Phi = \frac{2V_o}{\pi}\left[\tan^{-1}(G(r,z))\right] - \frac{\rho}{2\epsilon_o} z^2 + kz \qquad (3.32)$$

where

$$G(r,z) = \left[2^{1/2} a \left\{ r^2 - a^2 + \left[(r^2-a^2)^2 + 4a^2 z^2\right]^{1/2}\right\}^{-1/2}\right] \qquad (3.33)$$

and

$$k = \left\{ - \frac{2V_o}{\pi}\left[\tan^{-1}\left[G(r=0,d)\right]\right. + \frac{\rho}{2\epsilon_o} d^2 \right\} \frac{1}{d} \qquad (3.34)$$

a = radius of the disc

d = sheath thickness

Poisson's equation was solved satisfying the boundary conditions of the idealized sheath model. That is, the potential at the surface of the probe equals the probe potential, V_o, while at the sheath edge, the potential equals zero.

The electric field which is confined within the sheath is evaluated by taking the gradient of the potential.

$$\bar{E} = -\nabla \Phi = -\frac{\partial \phi}{\partial r}\hat{r} - \frac{\partial \phi}{\partial z}\hat{z}$$

Hence; the r-component of the electric field is given in equation (3.35).

$$E_r = \frac{2}{\sqrt{2}} \frac{V_o}{\pi a} \frac{\left[1 - a^2 (a^4 + 4b^2 z^2)^{-1/2}\right] r}{\left[-a^2 + (a^4 + 4b^2 z^2)^{1/2}\right]^{1/2}} \qquad (3.35)$$

The z-component of the electric field can be written:

$$E_z = -a^2 G(r, z) \frac{\left[r^2 - a^2 + \sqrt{(r^2-a^2)^2 + 4a^2z^2}\,\right]^{-1}}{\left[r^2-a^2) + 4a^2z^2\right]^{1/2}} + \frac{\rho z}{\epsilon_o} - k \quad . \quad (3.36)$$

By restricting the analysis to the thin sheath and small angle approximations, equations (3.35) and (3.36) can be simplified.

$$E_r = \frac{2V_o}{\pi a^3} \; z \, . \, r \qquad (3.37)$$

$$E_z = \frac{V_o}{\pi a} + \frac{\rho z}{\epsilon_o} - k \qquad (3.38)$$

It should be pointed out that as an ion traverses the sheath the deflection, Δr, will be small when compared to the sheath thickness, d. Hence, the radial component of electric field will vary only with z and can be written:

$$E_r = \frac{2V_o r_o}{\pi a^3} z \qquad (3.39)$$

where r_o is the distance from the z-axis that the ion enters the sheath.

Neglecting the effect of the earth's magnetic field on the motion of a collected particle, the electric field will contribute the only force acting on the particle. Hence the

Lorentz force equation can be written for the force acting on an ion in the z-direction:

$$\frac{mdV_z}{dt} = qE_z \tag{3.40}$$

where V_z = ion velocity in the z-direction

m = the mass of the collected ion

q = electron charge.

By substituting in for the electric field, equation (3.41) can be written.

$$\frac{d^2z}{dt^2} = \frac{q}{m}\left[\frac{\rho z}{\epsilon_o} + \frac{V_o}{\pi a} - k\right] \tag{3.41}$$

Rewriting equation (3.41),

$$\frac{1}{2}\left(\frac{dz}{dt}\right)^2 = \frac{q}{m} \quad \left(\frac{\rho z}{\epsilon_o} + \frac{V_o}{\pi a} - k\right)dz + \frac{C_o}{2} \tag{3.42}$$

and solving for the z-component of ion velocity yields:

$$V_z = \left[\frac{2q}{m}\left(\frac{\rho z^2}{2\epsilon_o} + \left(\frac{V_o}{\pi a} - k\right)z\right) + C_o\right]^{1/2} \quad . \tag{3.43}$$

The constant of integration, C_o, is evaluated by applying the boundary condition that $V_z = -V_R$ at $z = d$.

$$C_o = V_R^2 - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right] \tag{3.44}$$

By expanding equation (3.43), it becomes possible to integrate the expression term by term and obtain a relation between time and the z-component of position. Therefore f(z) is defined to be

$$f(z) = \left[\frac{2q}{m}(\frac{\rho z^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)z) + C_o\right]^{-1/2} \tag{3.45}$$

which is expanded in a Maclaurin series giving equation (3.46).

$$f(z) = f(0) + zf'(0) + \frac{z^2}{2}f''(0) + \ldots \tag{3.46}$$

Equation (3.43) can be rewritten in the expanded form.

$$dt = \left[f(0) + zf'(0) + \frac{z^2}{2}f''(0) + \ldots\right] dz \tag{3.47}$$

Equation (3.47) can be integrated term by term between the appropriate limits.

$$\int_o^t dt = \int_d^z \left[f(0) + zf'(0) + \frac{z^2}{2}f''(0) + \ldots\right] dz \tag{3.48}$$

For the thin sheath approximation it is necessary to consider only the first term of the expansion. Thus, an expression for t in terms of the position, z, can be written.

$$t = -\left[V_R^{\,2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + \left(\frac{V_o}{\pi} - k\right)d\right]\right]^{-1/2} \cdot (z-d) \tag{3.49}$$

Equation (3.49) can be solved for z, yielding an equation for z as a function of the time.

$$z(t) = -\left[V_R^{\,2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + \left(\frac{V_o}{\pi} - k\right)d\right]\right]^{1/2} t + d \tag{3.50}$$

By substituting equation (3.50) into equation (3.39) an expression for the radial component of the electric field acting on the particle as a function of time is determined.

$$E_r = \frac{2V_o}{\pi a^3} r_o \left\{ d - \left[V_R^{\,2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + \left(\frac{V_o}{\pi} - k\right)d\right]\right]^{1/2} t \right. \tag{3.51}$$

Using the Lorentz equation, the radial force acting on a collected ion can be written:

$$m\frac{d^2 r}{dt^2} = qE_r \quad . \tag{3.52}$$

Substituting equation (3.51) in for the radial component of the electric field

$$m\frac{d^2r}{dt} = \frac{2qV_o r_o}{\pi a^3}\left\{d - \left[V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right]\right]^{1/2} t\right\} \quad (3.53)$$

By separating the variables, equation (3.53) can be integrated directly to obtain the radial component of the particle's velocity.

$$\frac{dr}{dt} = \frac{2qV_o r_o}{m\pi a^3}\left\{d\cdot t - \left[V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right]\right]^{1/2}\frac{t^2}{2}\right\} + C_1 \quad (3.54)$$

The constant of integration, C_1, is evaluated at time equal to zero when the ion has no radial component of velocity. Remembering at time equal to zero, the ion is at the edge of the sheath. Solving for the constant at time equal to zero yields $C_1 = 0$. Integrating the radial component of the ion velocity yields the radial component of displacement. The limits on t are from time equal to zero to time equal to T_P, the time required for an ion which enters the sheath to be collected. T_P can be evaluated by substituting z=0 into equation (3.49).

$$T_P = \left[V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right]\right]^{-1/2} d \quad (3.55)$$

The limits on r are from $r=r_o$ occurring at time equal to zero to r=R occurring at time equal to T_P.

$$\int_{r_o}^{R} dr = \frac{2qV_o r_o}{m\pi a^3} \int_o^{T_P} \left\{ d \cdot t - \left[V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right]\right]^{1/2} \right.$$

$$\left. x \frac{t^2}{2} \right\} dt \qquad (3.56)$$

Performing the necessary integration:

$$R - r_o = \frac{2qV_o r_o}{m\pi a^3} \left\{ d \frac{T_P^{\ 2}}{2} - \left[V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi} - k)d\right]\right]^{1/2} \frac{T_P^{\ 3}}{6} \right\} \qquad (3.57)$$

Substituting in for T_P into equation (3.57),

$$R - r_o = \frac{2qV_o r_o d^3}{3m\pi a^3} \left\{ V_R^{\ 2} - \frac{2q}{m}\left[\frac{\rho d^2}{2\epsilon_o} + (\frac{V_o}{\pi a} - k)d\right]\right\}^{-1} . \qquad (3.58)$$

For the thin sheath and small angle approximation k is nearly equal to:

$$k \approx -\frac{V_o}{d} + \frac{2V_o}{\pi a} + \frac{\rho d}{2\epsilon_o} . \qquad (3.59)$$

When this expression for R is substituted back into equation (3.58) and rearranging terms yields:

$$\frac{\Delta R}{r_o} = \frac{R - r_o}{r_o} = -\frac{2q}{3m\pi} \left(\frac{d}{a}\right)^3 \left[\frac{V_R^{\ 2}}{V_o} - \frac{2q}{m}\left(1 - \frac{d}{\pi a}\right)\right]^{-1} . \qquad (3.60)$$

By knowing d/a and V_R^2/V_o, the ratio of particle deflection, ΔR, to r_o can be determined using equation (3.60). It should be noted that the deflection is strongly dependent on the ratio $(d/a)^3$.

3.4 Error Analysis of the Planar-Disc Probe

The positive ion current to a planar-disc probe is comprised of two components as was the ion current to the spherical probe. The ram current as explained in sections (2.1) and (2.2) is one component of the current. The other component is due to the sheath effects, and is caused by the ions which are deflected to the collector plate as they traverse the sheath.

It can be shown that the sheath current, I_s, is given by:

$$I_s = q\alpha N V_R \left\{ \left[\left(\frac{\Delta R}{r_o} + 1\right) r_o \right]^2 \pi - \pi r_o^2 \right\} \qquad (3.61)$$

where $\Delta R/r_o$ is given by equation (3.60), and r_o is the radius of the collector. The total ion current can then be written

$$I_T = qN\alpha V_R \left[\left(\frac{\Delta R}{r_o} + 1\right) r_o \right]^2 \pi \quad . \qquad (3.62)$$

An effective area can be defined as in the case of the spherical probe. Hence, equation (3.62) can be rewritten:

$$I_T = qN\alpha V_R A_{eff} \qquad (3.63)$$

where

$$A_{eff} = \left[\left(\frac{\Delta R}{r_o} + 1 \right) r_o \right]^2 \pi \qquad (3.64)$$

The error introduced by the sheath current is given by equation (3.65).

$$\text{Per cent error} = E = \frac{A_{eff} - A}{A_{eff}} \times 100 \text{ per cent} \qquad (3.65)$$

Substituting the expressions in for the respective areas, equation (3.65) becomes:

$$E = \left[1 - \left(\frac{\Delta R}{r_o} + 1 \right)^{-2} \right] \times 100 \text{ per cent} \qquad (3.66)$$

where $\frac{\Delta R}{r_o}$ is given by equation (3.60).

CHAPTER IV

APPLICATION OF THE THEORY

4.1 Limiting Cases of the Error

A comparison can now be made between the spherical and planar-disc probes. This analysis should help to illustrate the relative advantage in using a planar probe to one of a curved geometry. The two limiting cases of special interest are:

1. $V_R^2/V_o \rightarrow 0$

2. $V_R^2/V_o \rightarrow \infty$.

Case (1) in reality never exists since ram probe theory requires that the velocity of the probe be much greater than the thermal ion velocity. Case (1) does, however, help to illustrate what happens when the probe velocity is small and the probe potential is high. This condition actually sets an upper bound on the error due to the sheath current. For $V_R^2/V_o = 0$, equation (3.27) reduces for the spherical probe giving a deflection ratio equal to

$$\frac{\Delta x}{x_o} = \frac{d}{a+d} \ . \tag{4.1}$$

Similarly equation (3.60) for the planar-disc probe simplifies to

$$\frac{\Delta R}{r_o} = \frac{1}{3\pi}\left(\frac{d}{a}\right)^3 . \tag{4.2}$$

The deflection ratios for both geometries are given in Table (4.1) with the associated error, E, for various values of d/a. As would be suspected, the error increases with increasing d/a. An absolute upper limit can be placed on the error introduced by the sheath current when d/a attains its maximum value. For both ram probe theory, and this analysis to hold, it is necessary that the ratio d/a is small. Therefore it will be assumed that d/a is always less or equal to 0.5. Thus, by Table (4.1), an upper bound for the error associated with the sheath current is 44 per cent for the spherical probe to only 1.5 per cent for the planar-disc probe.

Case (2), $V_R^2/V_o \rightarrow \infty$, is applicable whenever the vehicle potential goes to zero, but may also be applied for extremely high vehicle velocities. As V_R^2/V_o becomes large, the deflection ratio for both the sphere and the disc become small. When $V_R^2/V_o = \infty$, both deflection ratios equal zero, and the errors associated with each are then zero. Thus, to minimize error it is of prime importance to keep the ratio of V_R^2/V_o as large as possible.

Figure 4.1 gives the deflection ratio for some typical values of V_R^2/V_o for both the spherical and planar-disc probe. From the deflection ratios, the error, E, is determined and plotted in Figure (4.2).

4.2 Application to the Mother-Daughter Ion Probe

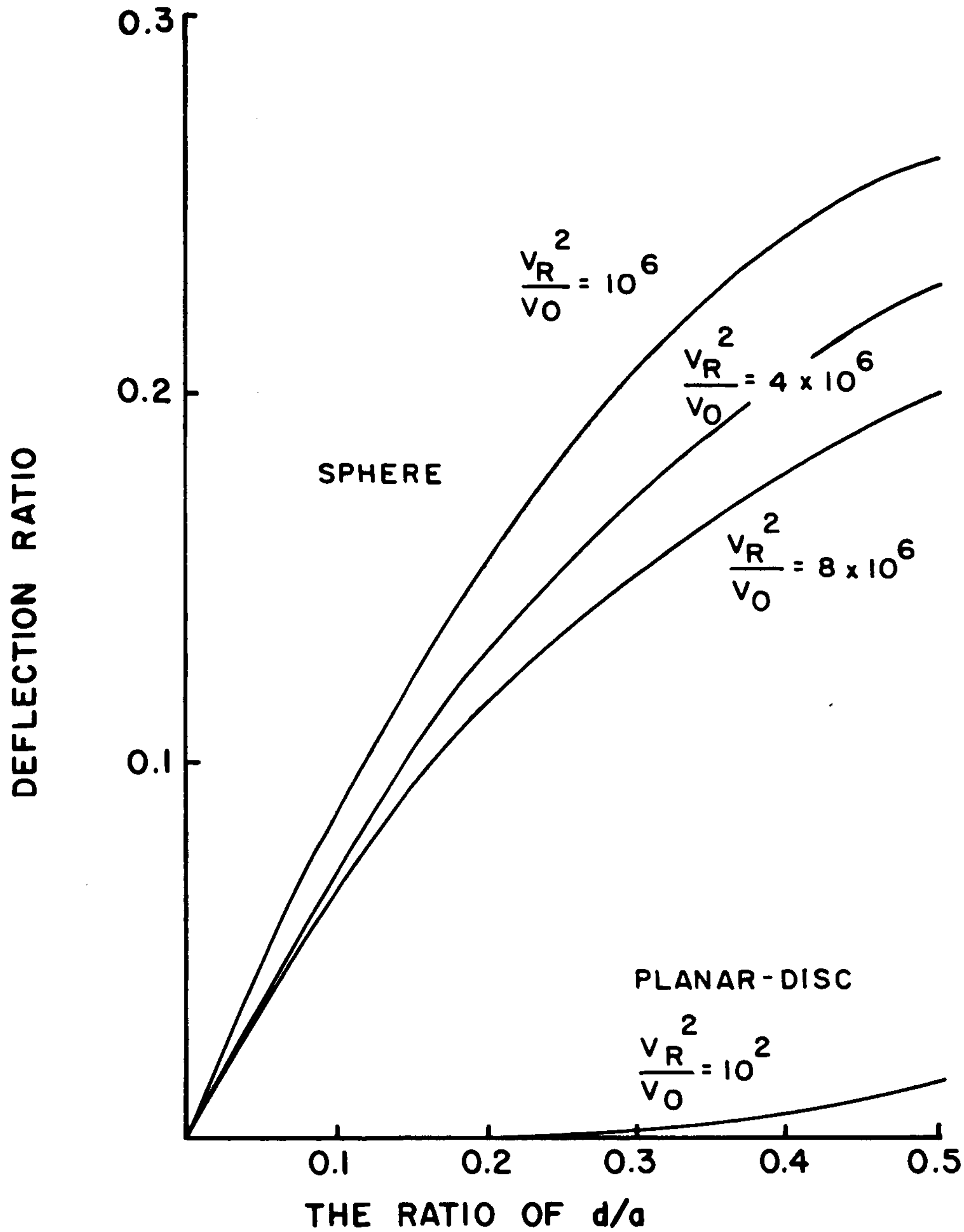
A planar-disc, retarding potential ion probe was flown on

MAXIMUM ERROR CONDITIONS FOR THE SPHERE

The Sphere with $V_R^2/V_o = 0$		
d/a	$\Delta x/x_o$	Error
0.1	0.09	
0.2	0.16	25%
0.3	0.23	33%
0.4	0.28	37%
0.5	0.33	44%

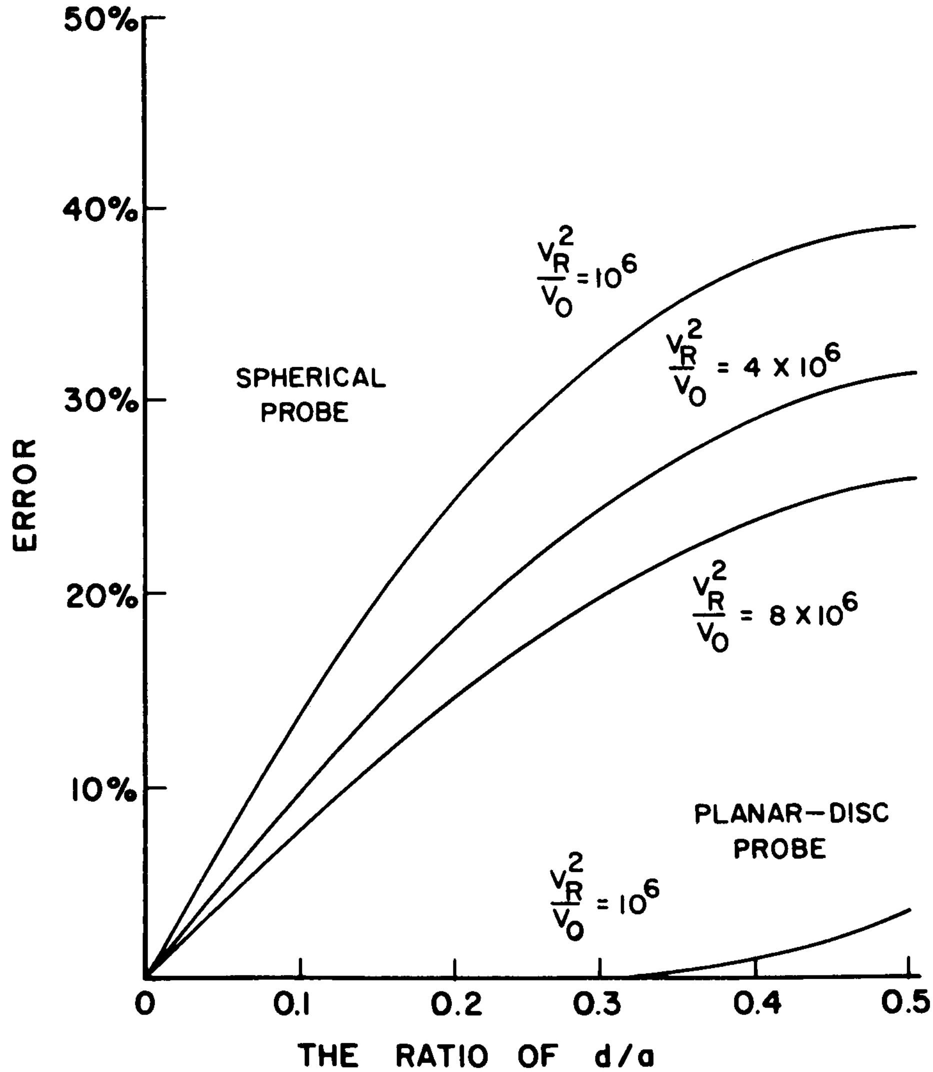
MAXIMUM ERROR CONDITIONS FOR THE DISC

The Planar Disc with $V_R^2/V_o = 0$		
d/a	$\Delta R/r_o$	Error
0.1	0.0010	Less Than
0.2	0.0080	1%
0.3	0.0027	
0.4	0.0064	
0.5	0.0125	1.5%



THE ION DEFLECTION FOR THE SPHERE AND DISC

FIGURE 4.1



THE SHEATH ERROR OF A SPHERE AND A PLANAR-DISC

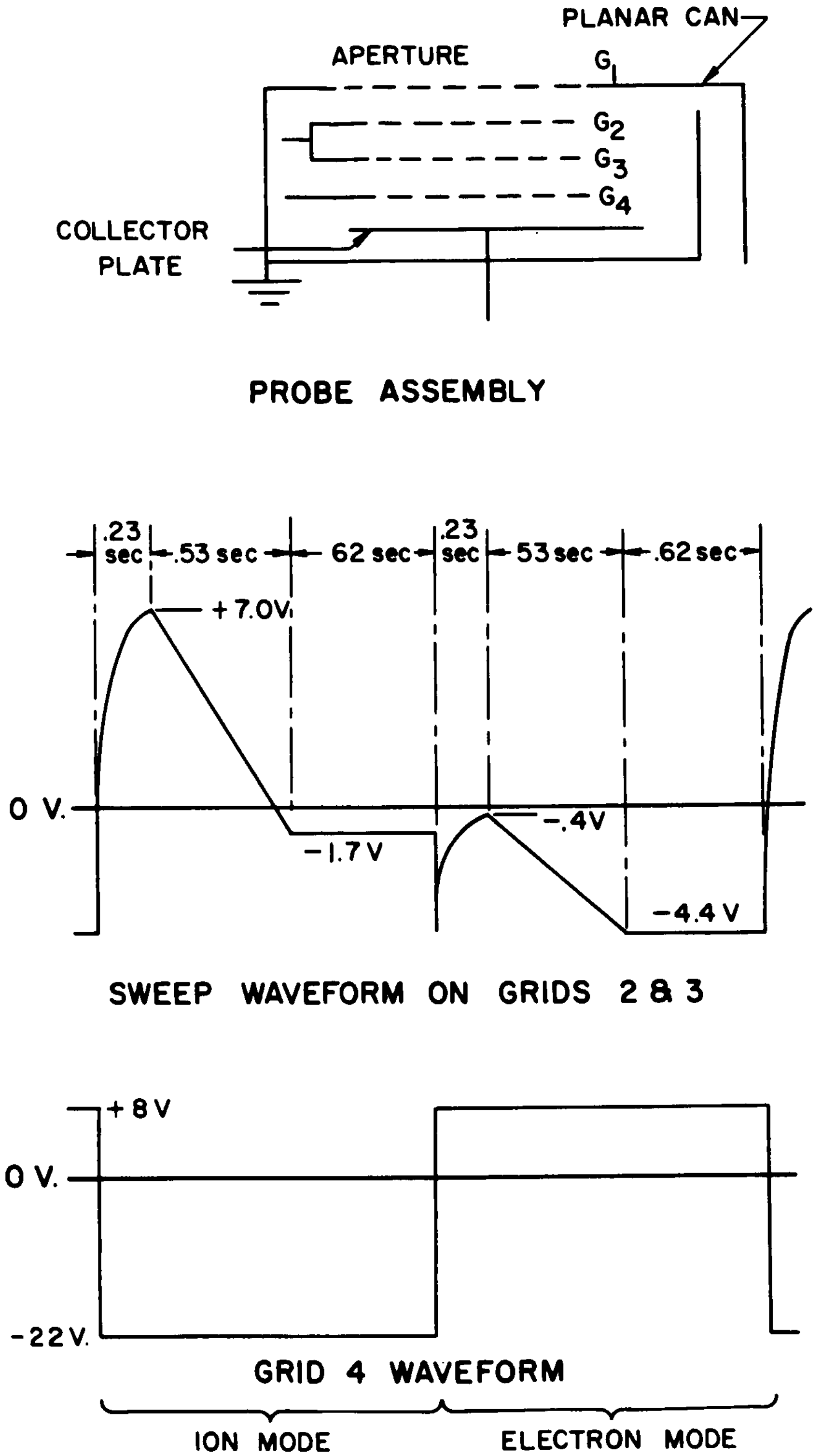
FIGURE 4.2

a Javelin rocket from Wallops Island, Virginia on May 19, 1965. This experiment was conducted in conjunction with the mother-daughter project which was a separating-payload propagation experiment for measuring electron densities.

The data reduction and analysis of the ion probe experiment was carried out by Douglas J. Hoffman (1966). The probe and associated electronics was designed by L. C. Hale and is described in a detailed report, L. C. Hale (1964).

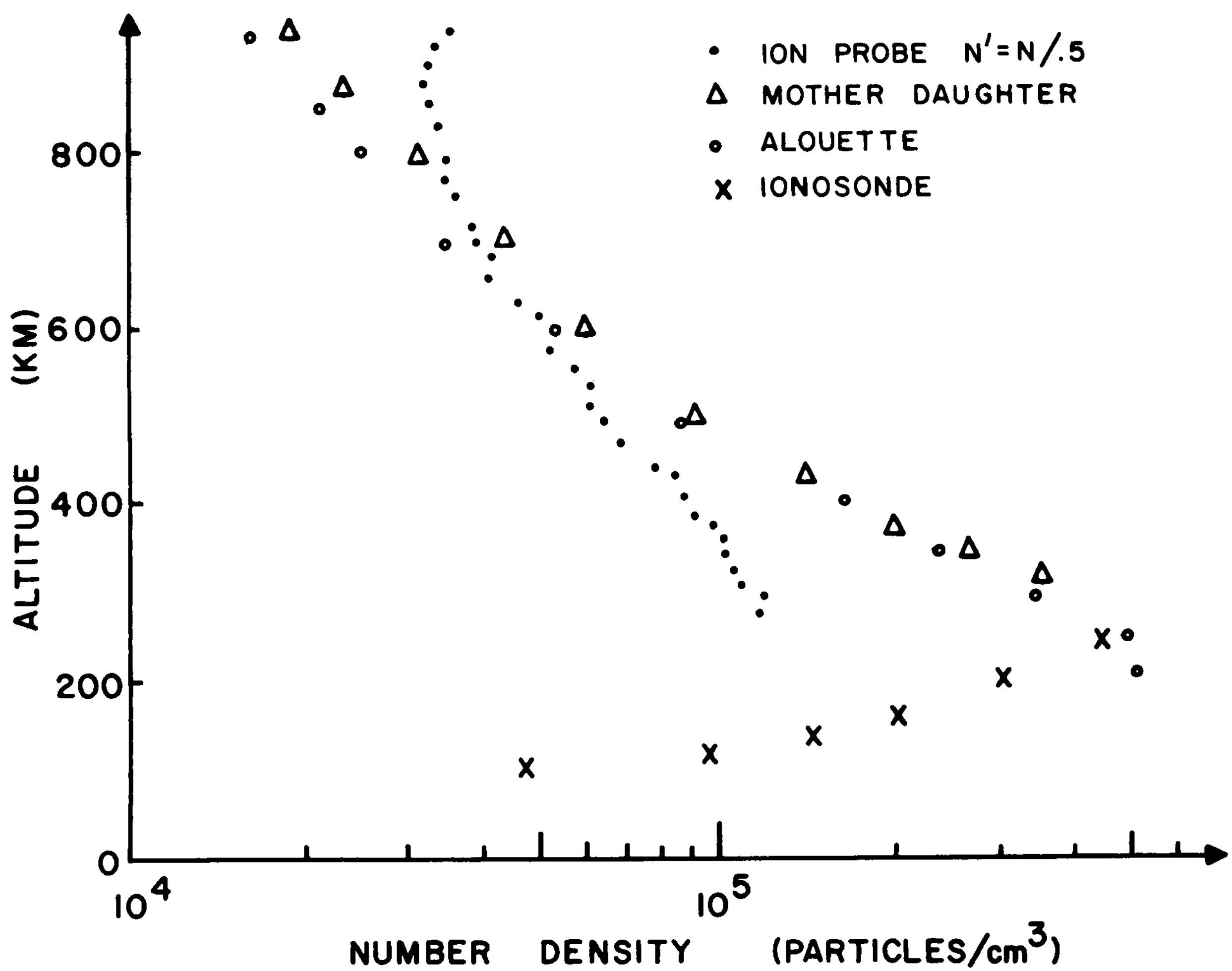
The probe operated in both the ion and electron modes. A set of four grids controls the operation of the probe, and is shown in Figure 4.3 with the associated waveforms. In the ion mode, a negative voltage suppresses photo-electrons and secondary emission from the collector plate. Both the collector and the planar-can including grid G_1 are held at vehicle potential. The linear sweep voltage which is applied to grids G_2 and G_3 establish a uniform potential.

Figure (4.4) gives Hoffman's reduce data for the ion density which was measured by the mother-daughter ion probe. Several other sources of ion density data are also plotted. The vehicle potential was experimentally determined as reported by Hoffman and remained relatively constant at about -1.5 volts throughout the flight. Using this information, the sheath thickness for the planar ion probe can be determined using equation (2.12). The results are shown in Figure 4.5 as a function of altitude. Dividing the sheath thickness by the probe radius a=3.8 cm, the



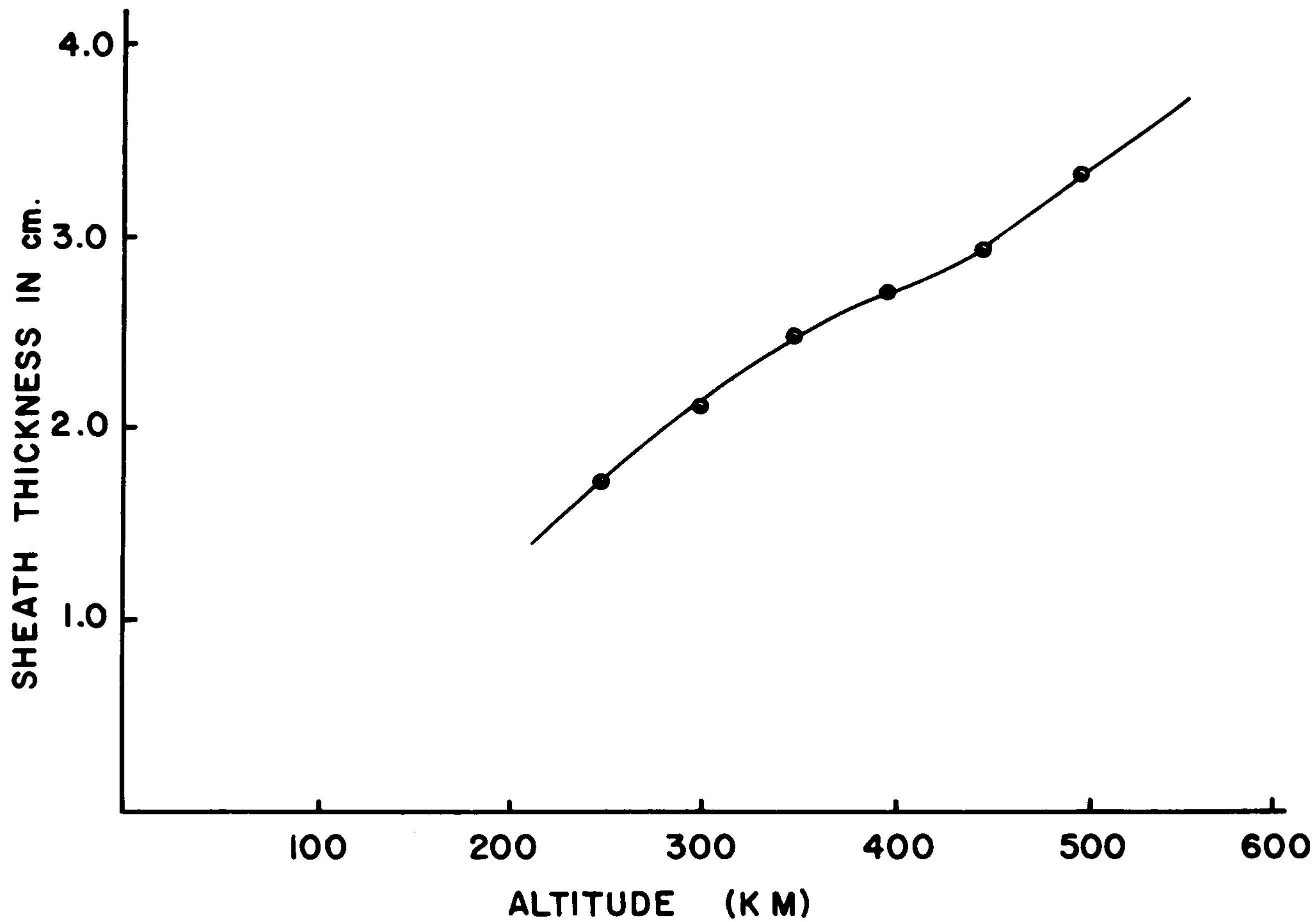
PROBE ASSEMBLY AND VOLTAGE WAVEFORMS

FIGURE 4.3



NORMALIZED ION DENSITY PROFILE

FIGURE 4.4



THE SHEATH THICKNESS OF M-D ION PROBE

FIGURE 4.5

ratio of d/a is found and given in Table (4.2).

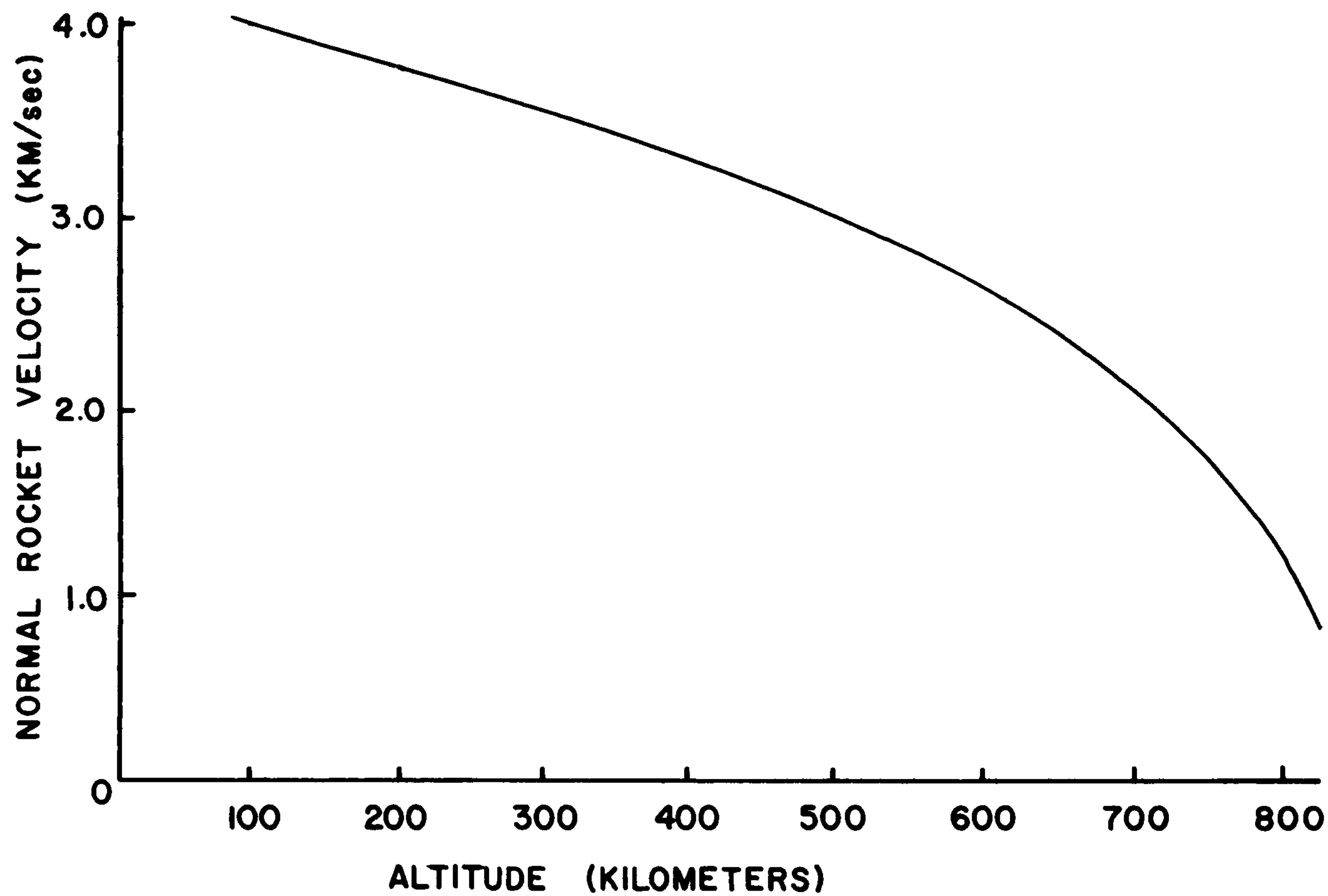
Figure (4.6) gives, V_R, the component of rocket velocity which is normal to the face of the probe, for the ascent of the NASA Javelin 8.29. Since the vehicle potential equals -1.5 volts, the ratio of V_R^2/V_o is determined and given in Table (4.2).

Using equation (3.60), the deflection ratio $\Delta R/r_o$ is determined and given in Table (4.2). The deflection ratio substituted into equation (3.66) yields the error due to the presence of the ion sheath. This is given in Table (4.2) and plotted in Figure (4.7).

For altitudes below 500 km the error introduced by the sheath is less than 2 per cent. Above 500 km the analysis no longer is valid because of the large value of d/a. This is really no shortcoming of the analysis; however, since ram probe theory is no longer applicable when d/a approaches unity. This analysis illustrates that the operation of planar-disc is extremely insensitive to the presence of the ion sheath provided that the ratio d/a is ketp less than unity.

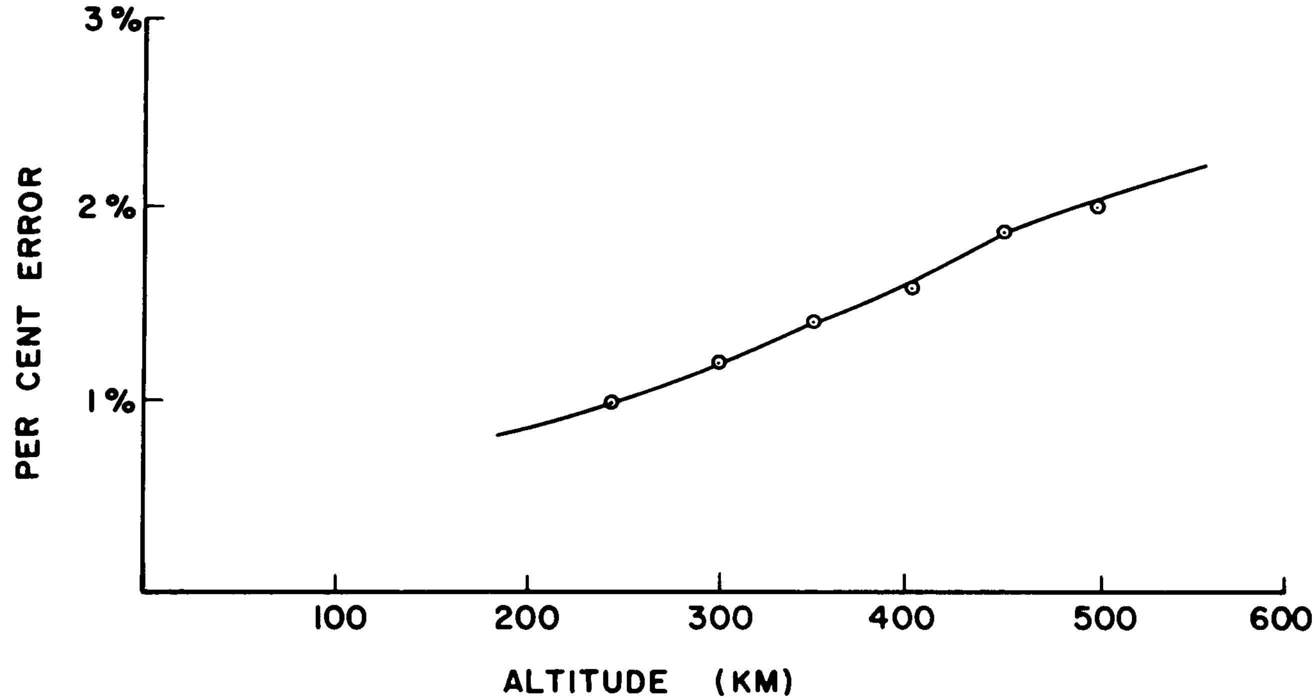
SUMMARY OF THE ERROR FOR THE MD ION PROBE

Altitude (km)	d(cm)	d/a	V_R^2/V_o	$\frac{\Delta R}{r_o}$	Error
250	1.7	0.45	$.73 \times 10^6$	0.0090	1%
300	2.1	0.55	$.71 \times 10^6$	0.0125	1.2%
350	2.5	0.66	$.69 \times 10^6$	0.0128	1.4%
400	2.7	0.71	$.60 \times 10^6$	0.0175	1.8%
450	2.9	0.76	$.59 \times 10^6$	0.0190	1.9%
500	3.3	0.87	$.58 \times 10^6$	0.0234	2.0%
700	4.4	1.15	$.27 \times 10^6$	~	~



NORMAL ROCKET VELOCITY

FIGURE 4.6



ERROR DUE TO SHEATH CURRENT FOR MD PROBE

FIGURE 4.7

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Geometry of the Probe

The analysis which was performed in Section (4.1) indicates the superiority of the planar-disc geometry to that of a sphere in the construction of high altitude ion probes. The operation of the planar-disc was extremely insensitive to the presence of the ion sheath when compared to the spherical probe operating under similar conditions. For a ratio d/a equal to 0.5 and the maximum error condition, $V_R^2/V_o = 0$, the error introduced by the presence of the sheath was shown to be only 1.5 per cent for the disc while 44 per cent for the sphere.

5.2 Error Minimization of a High Altitude Ion Probe

For altitudes below 500 km, the sheath error is less than 2.0 per cent. For altitudes above 500 km the sheath thickness approaches the size of the probe and no estimate of the error could be determined. If the ratio of d/a is not reduced it is doubtful that ion densities will be measured accurately by this method above 500 km because of the extreme complexity of the large sheath situation. For this reason it is suggested that an attempt be made to reduce the ratio d/a on future rocket flights.

For a vehicle potential of -1.5 volts, the sheath thickness at 1000 km reaches a maximum value of about 10 cm. By increasing the radius of the ion probe to 20 cm, the ratio of d/a would always be less than 0.5. Hence, ion densities could be measured to 1000 km with less than 2 per cent error associated to the sheath.

If payload capabilites prevent enlarging the probe, an alternate method of reducing the ratio of d/a can be employed. By applying a positive voltage to the face of the probe, the vehicle potential can be made to approach zero. As ϕ_o becomes small, the sheath thickness becomes small, and in turn the ratio of d/a become small. As the vehicle potential approaches zero, electrons will no longer be repelled from the collector plate. A grid biased sufficiently negative would be required to repel electrons away from the collector. A grid introduces an indeterminable amount of error due to its finite dimensions. For this reason, this method should only be used as a last resort in lowering the ratio of d/a.

5.3 Suggestions for Future Research

A rocket experiment should be designed and flown to an altitude of 1000 km utilizing the suggestions discussed in the previous section. From the experimental results it is hoped that the accuracy of a high altitude ion probe of this type will be substantiated.

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APPENDIX A

DETERMINING THE SHEATH THICKNESS FOR THE PLANAR-DISC PROBE

From equation (3.38), the z-component of the electric field can be written as:

$$E_z = \frac{V_o}{\pi a} + \frac{\rho z}{\epsilon_o} - k$$

where

$$k \sim -\frac{V_o}{d} + \frac{2V_o}{\pi a} + \frac{\rho d}{2\epsilon_o}$$

for a thin sheath. By Gauss's law the electric E_z at the edge of the sheath, z=d must equal zero.

$$E_z = V_o\left(\frac{1}{d} - \frac{1}{\pi a}\right) + \frac{\rho}{\epsilon_o}\left(\frac{d}{2}\right) = 0$$

Solving for d, yields:

$$d = \frac{\epsilon_o V_o}{\rho \pi a} + \sqrt{\left(\frac{2\epsilon_o V_o}{\pi a \rho}\right)^2 - \frac{2\epsilon_o V_o}{\rho}}$$